Joint Scheduling and Multiflow Maximization in Wireless Networks

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Maximum Multiflow Problem: Background

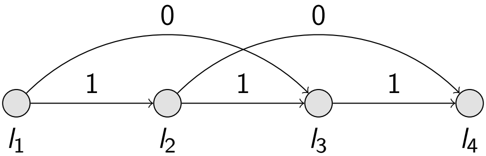
- We consider the **maximum multiflow** problem.
- To maximize the total (or concurrent) flow values between multiple sources and sinks, we usually have two steps:
  1. Model the wireless interference as a conflict graph to compute the scheduling rates (Jain et al., 2005);
  2. Find the maximum throughput supported by achievable scheduling rate.
- However, the first step is NP-hard and also difficult to approximate (Baker, 1994).
- The MMF problem is NP-hard even in simple settings (Wan, 2009).
- Existing joint optimization methods usually focus on either *approximate* solutions or *resitricted* networks.

Maximum Multiflow Problem: Our Contribution

- We investigate the exact-optimal (rather than approximate) solutions for general multi-hop networks.
- We *jointly* calculate the maximum multiflow and the scheduling rates by employing a *decomposition* method.
- Usually only a small subset of the scheduling rate region is needed.
- We prove the output solution is optimal.
- We consider the most general setting: *multiple multicast* with network coding (Ahlswede et al., 2000) allowed.

Network Model

- The network is modeled by $\mathcal{N} = (\mathcal{V}, \mathcal{L}, \mathcal{I}, D)$, where
  - $\mathcal{V}$ is the *node set*;
  - $\mathcal{L} \subseteq \mathcal{V}^2$ is the *link set*; each link l is associated with a *collision set* $\mathcal{I}(l)$;
  - $D : \mathcal{L}^2 \to \mathbb{Z}$ is a link-wise delay matrix.
  - We assume each link has a unit bandwidth and allow parallel links between nodes.
- $(\mathcal{L}, \mathcal{I}, D)$ can form a weighted, directed graph $\mathcal{N}$ where (l, l') is an edge if $l' \in \mathcal{I}(l)$, and $D(l, l')$ is the weight.
- For example, for a 4-link line network with unit delay and 1-hop interference model, we can draw:



Scheduling Rate Region

- Assuming time is slotted, when link l is active at time slot t and link $l' \in \mathcal{I}(l)$ is active at time slot $t + D(l, l')$, a *collision occurs*.
- We use a binary matrix $S : \mathcal{L} \times \mathbb{N} \to \{0, 1\}$ to specify a *schedule*: $S(l, t) = 1$ indicates that link l is active in time slot t.
- $S(l, t)$ has collision if $S(l, t) = S(l', t + D(l, l')) = 1$ for some $l' \in \mathcal{I}(l)$.
- S is collision free if $S(l, t)$ is collision free for all (l, t).
- For a collision-free schedule S and a link l, the link rate is

$$R_S(l) = \lim_{T\to\infty} \frac{1}{T} \sum_{t=0}^{T-1} S(l, t). \tag{1}$$

- $R_S = (R_S(l), l \in \mathcal{L})$ is the *rate vector* of S, and it is *achievable* if there is a collision-free schedule achieves S it.
- The collection $\mathcal{R}(\mathcal{N})$ of all the achievable rate vectors is called the *scheduling rate region*.

Problem Setting

- We now define the maximum multiflow (MMF) problem.
- We say $F = (F(l) \in \mathbb{N}_{\geq 0} : l \in \mathcal{L})$ is a valid flow from source $s \in \mathcal{V}$ to sink $t \in \mathcal{V}$ w.r.t. R if
  - $0 \leq F(l) \leq R(l)$ for all $l \in \mathcal{L}$;
  - $\sum_{l \in \mathrm{In}(v)} F(l) = \sum_{l \in \mathrm{Out}(v)} F(l)$ for all $v \in \mathcal{V} \setminus \{s, t\}$.
- Flow $\sum_{l \in \mathrm{Out}(s)} F(l)$ out of s equals to $\sum_{l \in \mathrm{In}(t)} F(l)$ into t, and it is called the *value* of F, denoted by $\mathrm{val}(F)$.
- **MMF problem**: Consider a source s_i multicasts a message to nodes in the set $\mathcal{D}_{s_i} = \{t_{i,1}, \ldots, t_{i,k_i}\}$ (with network coding) at a rate

$$v_i = \min_j \mathrm{val}(F_{i,j}),$$

  where $F_{i,j}$ is a flow from s_i to $t_{i,j}$ for each j.
- Fix rate vector R. Link l has to accommodate all k flow requirements simultaneously, i.e., $\sum_{i=1}^{k} \max_j F_{i,j}(l) \leq R(l)$ for all $l \in \mathcal{L}$.

Problem Setting

- LP-MMF($\tilde{\mathcal{R}}$): maximize the sum of rates of multicasting k sources:

$$\begin{aligned}
\text{maximize} \quad & \sum_{i=1}^{k} v_i && (2)\\
\text{subject to} \quad & F_{i,j} \text{ is a valid flow}, \forall i \in [k], j \in [k_i], \\
& \sum_{l \in \mathrm{Out}(s_i)} F_{i,j}(l) = \sum_{l \in \mathrm{In}(t_{i,j})} F_{i,j}(l) = v_i, \ \forall i \in [k], j \in [k_i], \\
& G_i(l) \geq F_{i,j}(l), \ \forall l \in \mathcal{L}, i \in [k], j \in [k_i], \\
& \sum_{i=1}^{k} G_i(l) \leq R(l), \ \forall l \in \mathcal{L}, && (3)\\
& R \in \tilde{\mathcal{R}},
\end{aligned}$$

where $[k] := \{1, \ldots, k\}$, $k, k_i \in \mathbb{N}_+, \forall i$.

- It takes polytope $\tilde{\mathcal{R}}$ as an input.
- (3) gives a dual vector μ; $\mu(l)$ indicates how sensitive the optimization objective is to the rate constraint $R(l)$.

Algorithm

- Our joint algorithm computes the optimal MMF while only requires a subset of the scheduling rate region.
- We attach $a_i \geq 0$ to link l_i, and with $\mathbf{a} = (a_i)_{i=1,\ldots,|\mathcal{L}|}$ we solve

$$\mathrm{argmax}_{R\in\mathcal{R}}\langle \mathbf{a}, R\rangle, \tag{4}$$

- This corresponds to a weighted maximal independent set problem

$$\begin{aligned} \text{ILP:} \quad & \text{maximize} \quad \sum_{i=1}^{|\mathcal{L}|} a_i S(l_i) \\ & \text{subject to} \quad S(l_i) + S(l_j) \leq 1, \quad \forall\, l_i, l_j : \; l_j \in \mathcal{I}(l_i). \end{aligned}$$

- The solution gives us an achievable rate vector, i.e., a vertex of $\mathcal{R}$.
- The ILP can have high complexity; we deal with this later.
- Based on (4), we iteratively search the MMF and the scheduling rates.

Algorithm

Algorithm 1

Input: a network $(\mathcal{V}, \mathcal{L}, \mathcal{I})$ **Output**: maximum multiflow v

Start with any rate vector $R_1 \in \mathcal{R}$, $\mathcal{R}_1 \leftarrow \{R_1\}$
for $i = 1, 2, \ldots$ **do**
  Run $v_i \leftarrow$LP-MMF$(\mathcal{R}_i)$ and obtain the dual vector μ_i
  Run ILP to find $R_{i+1} \leftarrow \mathrm{argmax}_{R \in \mathcal{R}} \langle \mu_i, R \rangle$
  $\mathcal{R}_{i+1} \leftarrow \mathrm{conv}\left(\mathcal{R}_i \cup \{R_{i+1}\}\right)$
  if $\langle \mu_i, R_{i+1} \rangle = \max_{R \in \mathcal{R}_i} \langle \mu_i, R \rangle$ **then**
    return v_i
  end if
end for

From $i = 1$, in each iteration, the algorithm works as follows:

1. We start with a known subset of the scheduling region $\mathcal{R}_i$. E.g., we start with $\mathcal{R}_1 = \{R_1\}$.
2. By the ILP, we find a new rate vector R_{i+1} by

$$R_{i+1} = \operatorname{argmax}_{R \in \mathcal{R}} \langle \mu_i, R \rangle. \tag{5}$$

3. We update the subset by $\mathcal{R}_{i+1} = \operatorname{conv}\left(\mathcal{R}_i \cup \{R_{i+1}\}\right)$.
4. If $\langle \mu_i, R_{i+1} \rangle = \max_{R \in \mathcal{R}_i} \langle \mu_i, R \rangle$, the algorithm terminates and outputs the last optimal value of LP-MMF($\mathcal{R}_i$).

The algorithm can terminate quite quickly; the remaining question is whether the output value is optimal.

Algorithm: Proof Sketch of Optimality

1. Since the scheduling region is a polytope with finite number of vertices, the Algorithm 1 will eventually terminate.
2. $\mathrm{flow}(R)$ is the function that outputs the MMF w.r.t. a rate vector R.
3. $\mathcal{R}$ is the entire scheduling region and $\mathcal{R}_i$ is the subset until iteration i.
4. If the algorithm ends at iteration i', $\langle \mu_{i'}, R_{i'+1} \rangle = \max_{R \in \mathcal{R}_{i'}} \langle \mu_{i'}, R \rangle$.
5. Combining with $R_{i+1} = \mathrm{argmax}_{R \in \mathcal{R}} \langle \mu_i, R \rangle$, we have

$$\max_{R \in \mathcal{R}} \langle \mu_{i'}, R \rangle = \max_{R \in \mathcal{R}_{i'}} \langle \mu_{i'}, R \rangle. \tag{6}$$

6. Suppose the optimal R in LP-MMF($\mathcal{R}_{i'}$) is R^*.
7. We can verify R^* maximizes $\mathrm{flow}(R) - \langle \mu_{i'}, R \rangle$ for $R \in \mathbb{R}_{\geq 0}^{|\mathcal{L}|}$, hence

$$\begin{aligned} \mathrm{flow}(R^*) - \langle \mu_{i'}, R^* \rangle &\geq \mathrm{flow}(R') - \langle \mu_{i'}, R' \rangle \\ &\overset{(a)}{\geq} \mathrm{flow}(R') - \langle \mu_{i'}, R^* \rangle, \end{aligned}$$

   where (a) is by (6) and the R^* maximizes $\langle \mu_{i'}, R \rangle$ for $R \in \mathcal{R}_{i'}$.
8. Therefore R^* maximizes $\mathrm{flow}(R)$ for $R \in \mathcal{R}$.

Algorithm

- The algorithm requires integer linear programming, hence it does not have a polynomial time complexity in theory, which is expected since this problem is NP-hard.
- We use simulation on random networks to show the efficiency.

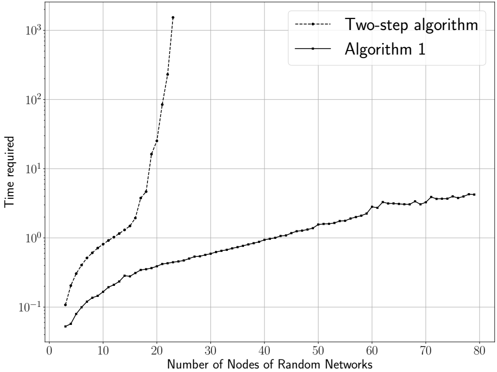
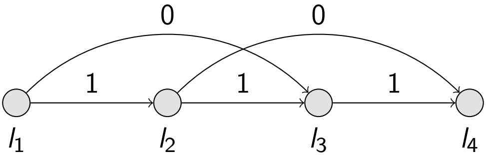


Figure: Results of 1-hop interference and multicast case of 1 source and 2 sinks.

Networks with Non-negligible Delays

- Recent results show that by utilizing non-negligible propagation delays, the scheduling region can be greatly improved (Yang et al., 2023).
- However, the scheduling problem suffers doubly-exponential complexity.
- We adopt a similar graphical formulation and extend our algorithm.
- We first review the scheduling problem.
- Recall for $|\mathcal{L}| = 4$ and unit delay and 1-hop interference, we have:



Networks with Non-negligible Delays

- For a collision-free schedule matrix S and integers $T \in \mathbb{N}_+$, $k \in \mathbb{Z}$, $S[T,k]$ denotes the submatrix of S consisting of columns $kT, kT+1, \ldots, (k+1)T-1$.

Definition (Scheduling Graph (Ma et al., 2021))

Given a network $\mathcal{N}$ and an integer $T > 0$, a *scheduling graph* $(\mathcal{M}_T, \mathcal{E}_T)$ satisfies: the vertex set $\mathcal{M}_T$ includes all the $|\mathcal{L}| \times T$ binary matrices A such that $A = S[T,0]$ for a certain collision-free schedule S. The edge set $\mathcal{E}_T$ includes all the vertex pairs (A,B) such that $A = S[T,0]$ and $B = S[T,1]$ for a certain collision-free schedule S.

- By choosing $T \geq \max_{l\in\mathcal{L}} \max_{l'\in\mathcal{I}(l)} |D(l,l')|$, the scheduling problem is searching all the cycles in the scheduling graph, which is NP-hard.

Networks with Non-negligible Delays

- To solve the MMF problem, the key is to have a function similar to $\operatorname{argmax}_{R\in\mathcal{R}}\langle \mathbf{a}, R\rangle$ that gives a rate vector fast.
- We hence define a *weighted* scheduling graph:

Definition

Given a weight vector $\mathbf{a} \in \mathbb{R}^{|\mathcal{L}|}$ and a scheduling graph $(\mathcal{M}_T, \mathcal{E}_T)$, a *weighted scheduling graph* $(\mathcal{M}_T, \mathcal{E}_T, w_\mathbf{a})$ is a directed, weighted graph such that: the vertex set is $\mathcal{M}_T$, and for each directed edge (v_1, v_2) in $(\mathcal{M}_T, \mathcal{E}_T)$, there is a weighed, directed edge (v_1, v_2) in $(\mathcal{M}_T, \mathcal{E}_T, w_\mathbf{a})$ with weight $w_\mathbf{a}(v_1, v_2) = \mathbf{a}^\intercal v_2 \mathbf{1}$.

- Given a vector $\mathbf{a}$, finding a vector that solves $\operatorname{argmax}_{R\in\mathcal{R}}\langle \mathbf{a}, R\rangle$ is equivalent to finding the maximum-mean-cycle in $(\mathcal{M}_T, \mathcal{E}_T, w_\mathbf{a})$.
- This can be solved with a polynomial complexity $\Theta(|\mathcal{M}_T|\,|\mathcal{E}_T|)$ by the Karp's algorithm (Karp, 1978).

Algorithm

Algorithm 2 Algorithm for Networks with Delays

Input: a network $\mathcal{N} = (\mathcal{V}, \mathcal{L}, \mathcal{I}, D)$
Output: maximum multiflow v

Start with any rate vector $R_1 \in \mathcal{R}$, $\mathcal{R}_1 \leftarrow \{R_1\}$
Construct $(\mathcal{M}_T, \mathcal{E}_T)$ from $\mathcal{N}$
for $i = 1, 2, \ldots$ **do**
  Run $v_i \leftarrow$LP-MMF$(\mathcal{R}_i)$ and obtain the dual vector μ_i
  Construct $(\mathcal{M}_T, \mathcal{E}_T, w_{\mu_i})$ by $(\mathcal{M}_T, \mathcal{E}_T)$ and μ_i
  Find maximum-mean-cycle in $(\mathcal{M}_T, \mathcal{E}_T, w_{\mu_i})$ and obtain $R_{i+1} \leftarrow \operatorname{argmax}_{R \in \mathcal{R}} \langle \mu_i, R \rangle$
  $\mathcal{R}_{i+1} \leftarrow \operatorname{conv}(\mathcal{R}_i \cup \{R_{i+1}\})$
  if $\langle \mu_i, R_{i+1} \rangle = \max_{R \in \mathcal{R}_i} \langle \mu_i, R \rangle$ **then**
    return v_i
  end if
end for

Networks with Non-negligible Delays

- We can similar prove the optimality of the output.
- We present simulation results on simple settings:

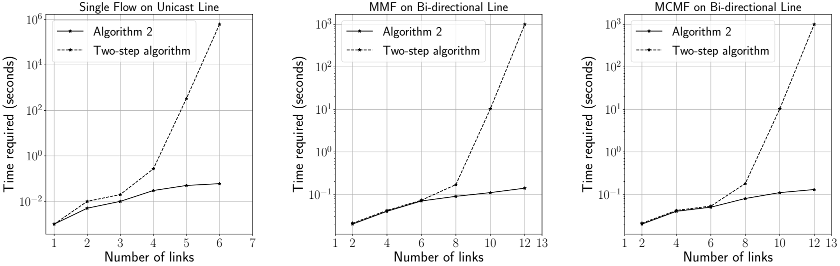


Figure: Line networks with unit delay

- For $\mathcal{N}_{4,1}^{D=1}$, the algorithm in (Ma et al., 2021) needs to search 7653 rate vectors, while we only need to find 2 of them for solving the MMF problem.

Conclusion

- We revisited the maximum multiflow problem that is NP-hard in theory.
- We provided an efficient joint algorithm that outputs optimal value in a finite number of iterations, while only requires a subset of the scheduling region.
- We studied the general multi-source multi-sink network with network coding.
- Our framework involves networks with non-negligible delay.
- We used simulation results to demonstrate our advantages.
- Future work: our maximum-mean-cycle algorithm could improve the calculation of the scheduling rate region (Ma et al., 2021; Yang et al., 2023).

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