Wireless Link Scheduling with Propagation Delays

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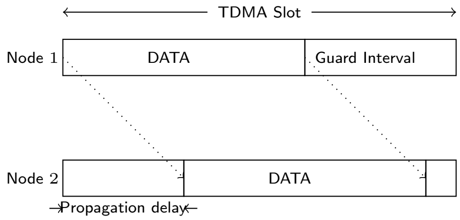
April 4, 2022

Guideline

- Finished works:
  1. Theoretical framework of link scheduling problems with propagation delays.
  2. Graphical characterization of network scheduling: cycles and maximal independent sets of graphs.
  3. Efficient algorithms exploiting nature of rate regions(target).
- Ongoing works:
  1. Exact characterization of rate regions of line networks.
  2. More systematic schemes to find inner and outer bounds of rate regions.
  3. Relationship with discrete(time-slotted) scheduling and continuous scheduling.
  4. Continuity of rate regions.

Propagation delay and Frame Size

	Underwater acoustic	Satellite	5G (Millimeter wave)	4G
Propagation delay	3.3s	0.12s	0.001ms	0.033ms
Frame size	10s	0.5s	10ms	10ms
Propagation speed	1.5km/s	3×10^5km/s	3×10^5km/s	3×10^5km/s
Transmission range	3km	3.6×10^4km	0.3km	10km



Early works

- Early works fight with propagation delay to improve framed scheduling and achieved very limited gains.
- Some researches take use of propagation delays in underwater MAC[1][2][3] and achieve higher throughput than framed scheduling.
- Some researches model it to optimization problems.

In an N-node wireless network where any two links can generate collisions to each other. Chitre[4] demonstrate thats:

- The optimal schedule is periodic .
- The upper bound of throughput with propagation delay is $N/2$.
- The upper bound of throughput with zero propagation delay is 1.

[1] Borja Peleato and Milica Stojanovic. "Distance aware collision avoidance protocol for ad-hoc underwater acoustic sensor networks". In: *IEEE Communications Letters* 11.12 (2007), pp. 1025–1027.

[2] Kurtis Kredo II, Petar Djukic, and Prasant Mohapatra. "STUMP: Exploiting position diversity in the staggered TDMA underwater MAC protocol". In: *INFOCOM 2009, IEEE*. IEEE. 2009, pp. 2961–2965.

[3] Hai-Heng Ng, Wee-Seng Soh, and Mehul Motani. "BiC-MAC: Bidirectional-concurrent MAC protocol with packet bursting for underwater acoustic networks". In: *OCEANS 2010 MTS/IEEE SEATTLE*. IEEE. 2010, pp. 1–7.

[4] Mandar Chitre, Mehul Motani, and Shiraz Shahabudeen. "Throughput of networks with large propagation delays". In: *IEEE Journal of Oceanic Engineering* 37.4 (2012), pp. 645–658.

Motivations

- **Rate region** can help understanding network resource allocation issues like rate control, power control and routing.
- In traditional framed scheduling, the rate region of link scheduling is taking the convex hull of all independent sets of a finite collision graph indicting the collision constraints between links.
- No existing works have given such a similar rate region in network with propagation delays.

Outlines

1. Network Model and Scheduling Rate Region
2. Rate Region Characterization
3. Algorithms for Cycles in Scheduling Graph
4. Rate Region of Line Networks

Network Model and Scheduling Rate Region

Link-wise network models

The network model is a weighted directed hypergraph: $\mathcal{N} = (\mathcal{L}, \mathcal{I}, D_{\mathcal{L}})$:

- $\mathcal{L}$ is the vertex set denoting links;
- $\mathcal{I}$ is the set of hyperedges denoting the collision relations among links;
- $D_{\mathcal{L}}$ is an $|\mathcal{L}| \times |\mathcal{L}|$ integer valued matrix with each entry $D_{\mathcal{L}}(l, l')$ specifying the weight from l to l'.

Links and Collision sets

- A *(communication) link* is a pair (s, r) where $1 \leq s \neq r \leq N$ indicating the transmitting and receiving nodes, respectively.
- Each link l is associated with a subset $\mathcal{I}(l)$ of $2^{\mathcal{L}}$, called the *collision set* of l. Each subset of links ϕ in the collision set $\mathcal{I}(l)$ may cause collision with l.

Binary collision

- When the collision set $\mathcal{I}(l)$ satisfies for any $\phi \in \mathcal{I}(l)$, $|\phi| = 1$, the collision set is called binary.
- When the collision set is binary, the network is binary model and $\mathcal{N} = (\mathcal{L}, \mathcal{I}, D_{\mathcal{L}})$ is a weighted directed graph.
- When the collision set is non-binary, the network is general model and $\mathcal{N} = (\mathcal{L}, \mathcal{I}, D_{\mathcal{L}})$ is a weighted directed hypergraph.

Linkwise delay matrix

- $D_{\mathcal{L}}(l, l') = D(\mathrm{s}_l, \mathrm{r}_l) - D(\mathrm{s}_{l'}, \mathrm{r}_l)$, where $D(i, j) \in \mathbb{Z}^+$ denotes the *signal propagation delay* from node i to node j.
- It is sufficient for us to check collisions using $D_{\mathcal{L}}$.

Network model of multihop line network $\mathcal{N}^{\text{line}}_{L,K}$

- The network has nodes indexed by $1, \ldots, L+1$.
- The link set
$$\mathcal{L} = \{l_i \triangleq (i, i+1), i = 1, \ldots, L\}.$$
- The collision set of link l_i is
$$\mathcal{I}(l_i) = \{l_j : j \neq i, |j - i - 1| \leq K\}.$$
- The link-wise delay matrix $D_{\mathcal{L}}$ of the L-length, K-hop collision line network has
$$D_{\mathcal{L}}(l_i, l_j) = D(i, i+1) - D(j, i+1) = 1 - |j - i - 1|.$$

Network model of multihop line network

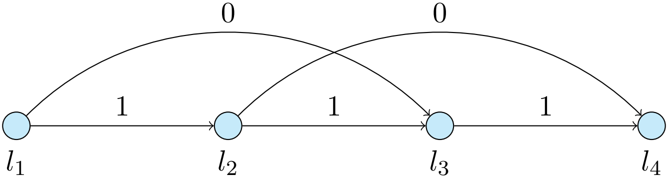
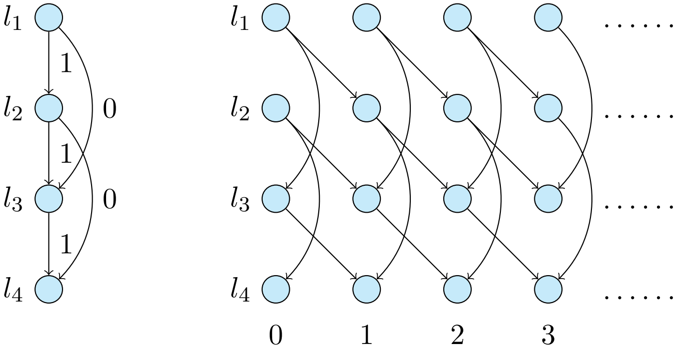


Figure: The graphical representation of $\mathcal{N}_{4,1}^{\text{line}}$.

Periodic graph

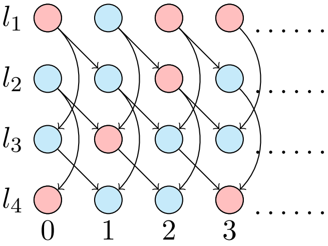
The (directed) periodic graph[5] $\mathcal{N}^{\infty}$ induced by network $\mathcal{N}$:



[5] James B Orlin. "Some problems on dynamic/periodic graphs". In: *Progress in Combinatorial Optimization*. W. R. Pulleyblank, Ed. Orlando, FL: Academic Press, 1984, pp. 273–293.

Collision-free schedule and independent set

A collision-free schedule on $\mathcal{N}$ indicates an independent set of $\mathcal{N}^\infty$ induced by $\mathcal{N}$.



$$S = \begin{matrix} l_1 \\ l_2 \\ l_3 \\ l_4 \end{matrix} \begin{bmatrix} 1 & 0 & 1 & 1 & \dots \\ 0 & 0 & 1 & 0 & \dots \\ 0 & 1 & 0 & 0 & \dots \\ 1 & 0 & 0 & 1 & \dots \end{bmatrix}.$$

Rate region

- For a network $\mathcal{N}$, denote for a collision free schedule S and a link l

$$R_S^{\mathcal{N}}(l) = \lim_{T\to\infty} \frac{1}{T} \sum_{t=0}^{T-1} S(l,t),$$

- We call $R_S^{\mathcal{N}} = (R_S^{\mathcal{N}}(l), l \in \mathcal{L})$ the *rate vector* of S.
- The collection $\mathcal{R}^{\mathcal{N}}$ of all the achievable rate vectors is called the rate region of $\mathcal{N}$.

Rate region

- For a network $\mathcal{N}$, the rate region $\mathcal{R}^{\mathcal{N}}$ can be achieved using collision-free, periodic schedules only.
- Define the *character* of the network $\mathcal{N}$ as:

$$D_{\mathcal{N}}^{*} = \max_{l \in \mathcal{L}} \max_{\phi \in \mathcal{I}(l)} \max_{l' \in \phi} |D_{\mathcal{L}}(l, l')|.$$

Rate region by subgraphs

- Define $\mathcal{N}^T$ as the subgraph of $\mathcal{N}^\infty$ with the vertex set $\mathcal{L} \times \{0, 1, \ldots, T-1\}$.
- Define $\mathcal{R}^{\mathcal{N}^T}$ as the convex hull of the rate vectors of all the independent sets of $\mathcal{N}^T$.

Theorem

For a network $\mathcal{N}$,

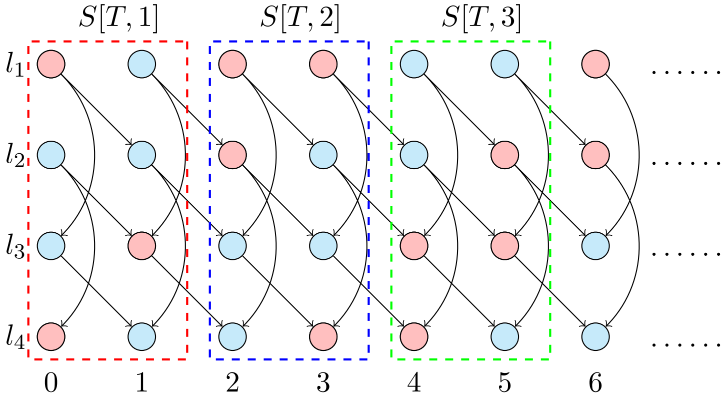
$$\mathcal{R}^{\mathcal{N}} = \mathrm{closure}\left(\cup_{T=1,2,\ldots} \frac{T}{T + D^*_{\mathcal{N}}} \mathcal{R}^{\mathcal{N}^T} \right),$$

where $\mathrm{closure}(\mathcal{A})$ is the closure of set A.

Rate Region Characterization

Conditional independence property

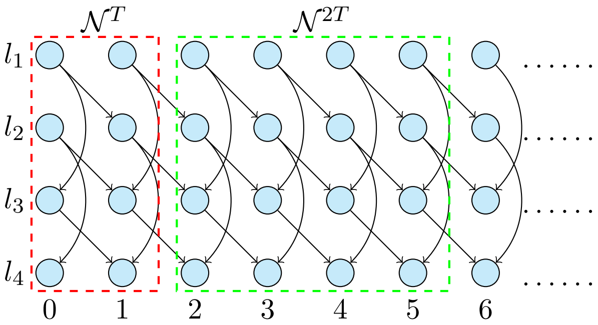
- Consider $S[T,k]$ as the submatrix of S with cols $kT,\ldots,(k+1)T-1$.
- Whether $S[T,2]$ is collision free is only affected by $S[T,1]$.
- Given $S[T,2]$, $S[T,1]$ and $S[T,3]$ do not affect the collision of each part when T is sufficiently large.
- We are dealing with **independent sets** of the graph.



Scheduling graphs

A *scheduling graph* is a directed graph denoted by $(\mathcal{M}_T, \mathcal{E}_T)$ defined as follows:

- $\mathcal{M}_T$ is the collection of all independent sets of $\mathcal{N}^T$.
- $\mathcal{E}_T$ is the collection of all independent sets of $\mathcal{N}^{2T}$.
- $T \geq D^*_{\mathcal{N}}$ for binary model and $T \geq 2D^*_{\mathcal{N}}$ for general model.

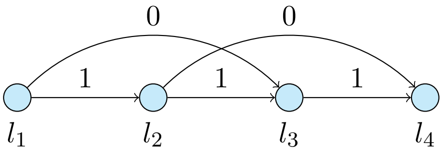


Scheduling graphs

For $\mathcal{N}_{4,1}^{\text{line}}$, $(\mathcal{M}_1, \mathcal{E}_1)$ has the vertex set $\mathcal{M}_1 = \{v_i, i = 0, 1, \ldots, 8\}$ where

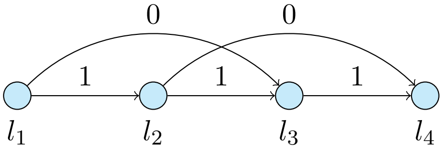
$$v_0 = \begin{bmatrix}0\\0\\0\\0\end{bmatrix}, v_1 = \begin{bmatrix}1\\0\\0\\0\end{bmatrix}, v_2 = \begin{bmatrix}0\\1\\0\\0\end{bmatrix}, v_3 = \begin{bmatrix}0\\0\\1\\0\end{bmatrix}, v_4 = \begin{bmatrix}0\\0\\0\\1\end{bmatrix},$$

$$v_5 = \begin{bmatrix}1\\0\\0\\1\end{bmatrix}, v_6 = \begin{bmatrix}1\\1\\0\\0\end{bmatrix}, v_7 = \begin{bmatrix}0\\1\\1\\0\end{bmatrix}, v_8 = \begin{bmatrix}0\\0\\1\\1\end{bmatrix},$$



The adjacency matrix of $\mathcal{E}_1$

$$
\begin{array}{c} \\ v_0 \\ v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \\ v_6 \\ v_7 \\ v_8 \end{array}
\begin{array}{c}
\begin{array}{ccccccccc} v_0 & v_1 & v_2 & v_3 & v_4 & v_5 & v_6 & v_7 & v_8 \end{array} \\
\begin{bmatrix}
1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\
1 & 1 & 0 & 1 & 1 & 1 & 0 & 0 & 1 \\
1 & 1 & 1 & 0 & 1 & 1 & 1 & 0 & 0 \\
1 & 1 & 1 & 1 & 0 & 0 & 1 & 1 & 0 \\
1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\
1 & 1 & 0 & 1 & 1 & 1 & 0 & 0 & 1 \\
1 & 1 & 0 & 0 & 1 & 1 & 0 & 0 & 0 \\
1 & 1 & 1 & 0 & 0 & 0 & 1 & 0 & 0 \\
1 & 1 & 1 & 1 & 0 & 0 & 1 & 1 & 0
\end{bmatrix}
\end{array}.
$$



Collision-free schedule and path in scheduling graph

- A path in scheduling graph $(\mathcal{M}_T, \mathcal{E}_T)$ is a sequence of binary matrices $A = (A_0, A_1, \ldots)$ where any $(A_i, A_{i+1}) \in \mathcal{E}_T$.
- A cycle is a path in scheduling graph, where the first and the last matrixs are the only repeated matrix.
- For general network model, when $T \geq 2D^*_{\mathcal{N}}$, a collision-free schedule S can be equivalent to a directed path in a schedule graph $(\mathcal{M}_T, \mathcal{E}_T)$. When the network is binary, $T \geq D^*_{\mathcal{N}}$ is sufficient.

Periodic schedule and cycles

- A collision-free, periodic schedule S forms a closed path in $(\mathcal{M}_T, \mathcal{E}_T)$.
- A closed path can be decomposed into a sequence of (not necessarily distinct) cycles[6].

Rate region can be characterized by cycles in scheduling graph.

[6] Petra Gleiss, Josef Leydold, and Peter Stadler. "Circuit bases of strongly connected digraphs". In: *Discussiones Mathematicae Graph Theory* 2.23 (2003), pp. 241–260.

Rate Region by cycles in scheduling graph

- Let $\mathrm{cycle}(\mathcal{G})$ be the set of cycles in a graph $\mathcal{G}$.
- Define

$$\mathcal{R}^{(\mathcal{M}_T,\mathcal{E}_T)} = \mathrm{conv}(\{R_C : C \in \mathrm{cycle}(\mathcal{M}_T,\mathcal{E}_T)\}),$$

- As $(\mathcal{M}_T,\mathcal{E}_T)$ is finite, $\mathrm{cycle}(\mathcal{M}_T,\mathcal{E}_T)$ is finite and hence $\mathcal{R}^{(\mathcal{M}_T,\mathcal{E}_T)}$ is a closed set.

Theorem

For a network $\mathcal{N}$ with general model and any integer $T \geq 2D^*_{\mathcal{N}}$, $\mathcal{R}^{\mathcal{N}} = \mathcal{R}^{(\mathcal{M}_T,\mathcal{E}_T)}$. When network $\mathcal{N}$ is binary, $T \geq D^*_{\mathcal{N}}$ is sufficient.

Rate Region by Cycles in Scheduling Graph

The rate region achieved by framed scheduling can be characterized using independent sets of directed graph $(\mathcal{L}, \mathcal{I})$. An independent set of directed graph $(\mathcal{L}, \mathcal{I})$ is equivalent with a 1-cycle in $(\mathcal{M}_1, \mathcal{E}_1)$.

Theorem

For a network $\mathcal{N} = (\mathcal{L}, \mathcal{I}, D_{\mathcal{L}})$*,*

$$\mathcal{R}^{(\mathcal{L},\mathcal{I})} = \mathrm{conv}(\{R_C : C \text{ is a } 1\text{-cycle in } (\mathcal{M}_1, \mathcal{E}_1)\})$$

and

$$\mathcal{R}^{(\mathcal{L},\mathcal{I})} \subset \mathcal{R}^{\mathcal{N}} \subset \mathcal{R}^{(\mathcal{M}_1,\mathcal{E}_1)}.$$

Algorithms for Cycles in Scheduling Graph

A straightforward approach

- Enumerating all the maximal independent sets of $\mathcal{N}^{2T}$ to calculate $\mathcal{M}_T$ and $\mathcal{E}_T$.
- Using Johnson's algorithm[7] to enumerate all the cycles in $(\mathcal{M}_T, \mathcal{E}_T)$.

[7] Donald B Johnson. "Finding all the elementary circuits of a directed graph". In: *SIAM Journal on Computing* 4.1 (1975), pp. 77–84.

A straightforward approach

Table: The computation time for enumerating all the cycles using Johnson's algorithm.

	$\mathcal{N}_{4,1}^{\text{line}}$	$\mathcal{N}_{5,1}^{\text{line}}$	$\mathcal{N}_{6,1}^{\text{line}}$
Number of vertices in $(\mathcal{M}_1, \mathcal{E}_1)$	9	15	25
Number of edges in $(\mathcal{M}_1, \mathcal{E}_1)$	56	144	357
Computation time	0.17s	5.5min	169.6h

A straightforward approach

Table: Run Johnson's algorithm for $(\mathcal{M}_1, \mathcal{E}_1)$ of $\mathcal{N}_{6,1}^{\text{line}}$. The table lists the cycle length range of a major fraction of cycles found during different running periods.

Running time period	$0 \sim 69.4$h	$69.5 \sim 111.1$h	$111.2 \sim 169.6$h
Cycle length range for 99% of cycles	$19 \sim 25$	$11 \sim 18$	$1 \sim 11$

- By our simulation results, most dominating vectors are achieved by short schedules, i.e., short cycles in $(\mathcal{M}_1, \mathcal{E}_1)$.

	L=1	L=2	L=3	L=4	L=5
K=1		1	1,4	1,4	1,4
K=2			1,4	1,4	1,4
K=3				1,3,4	
K=4					1,4,8,10

Algorithms for rate region

- Although *isomorphism* and *connectivity* can be used to reduce complexity in some special case.[8]
- In the case that $(\mathcal{M}_T, \mathcal{E}_T)$ is a complete graph, $\eta = 2^{|\mathcal{M}_T|}$, the computational complexity is double exponential in the network of size $|\mathcal{L}| \times T$.
- We want some algorithms more efficient and incremental.

[8] James B Orlin. "Some problems on dynamic/periodic graphs". In: *Progress in Combinatorial Optimization*. W. R. Pulleyblank, Ed. Orlando, FL: Academic Press, 1984, pp. 273–293.

Dominance properties

- $A \preccurlyeq B$ if all the entries of A are not larger than the corresponding entries of B at the same positions.
- For a set $\mathcal{A}$ with partial order $\succcurlyeq$, we write $\max_{\succcurlyeq} \mathcal{A}$ as the smallest subset $\mathcal{B}$ of $\mathcal{A}$ such that any element of $\mathcal{A}$ is dominated by certain elements of $\mathcal{B}$.
- A path(cycle) is said to be *maximal* if it is not dominated by any other path(cycle) of the same length in scheduling graph.

$$\begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix} \preccurlyeq \begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \end{bmatrix}$$

Dominance properties

We can represent $\mathcal{E}_T$ more concisely using $\mathcal{E}^* = \max_{\succcurlyeq} \mathcal{E}_T$, which is the collection of maximal independent sets of $\mathcal{N}^{2T}$. $\mathcal{E}^*$ can be represented by the adjacent matrix.

Example

$$\begin{array}{c c} & \begin{array}{cccc} v_5 & v_6 & v_7 & v_8 \end{array} \\ \begin{array}{c} v_5 \\ v_6 \\ v_7 \\ v_8 \end{array} & \begin{bmatrix} 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 \end{bmatrix} \end{array}.$$

Algorithm A

- Maximalpath: Enumerating the maximal paths incrementally.
- Path2cycle: Finding all cycles dominated by a path.

Maximalpath: example

Example

Consider the network $\mathcal{N}_{4,1}^{\text{line}}$. For $k = 1, \ldots, 4$, we list the number of maximal paths and the total number of length-k paths in $(\mathcal{M}_1, \mathcal{E}_1)$ in the following table:

	$k=1$	$k=2$	$k=3$	$k=4$
Number of length-k maximal paths	6	16	64	180
Number of length-k paths in $(\mathcal{M}_1, \mathcal{E}_1)$	56	363	2357	152633

Algorithm A

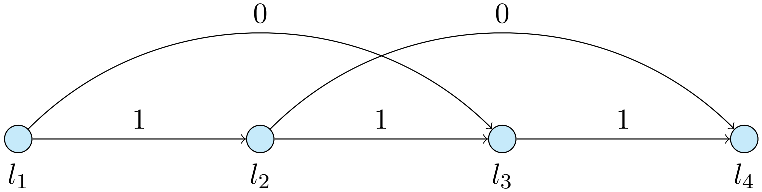
Table: Evaluations of the cost of enumerating all the cycles of given lengths with Algorithm A. (If the time cost is within 1 second, we write as 1 second)

Cycle length	1	2	3	4	5	6	7	8	9	10
$\mathcal{N}_{4,1}^{\text{line}}$	1s	1s	1s	1s	1s	1s	1s	2.76s	29.2s	-
$\mathcal{N}_{5,1}^{\text{line}}$	1s	1s	1s	1s	1s	1s	1.08s	16.1s	6.3min	2.4h
$\mathcal{N}_{6,1}^{\text{line}}$	1s	1s	1s	1s	1s	1s	10.1	4.37min	1.78h	2.29d

Algorithm B: Cycles associated with a maximal subgraph

- Using existing algorithm (e.g. depth-first search) to enumerate all the paths of $(\mathcal{M}^*, \mathcal{E}^*)$ up to length k.
- For each path, we use Path2cycle to find cycles dominated by the path.

Algorithm results: multihop line network



Algorithm results: multihop line network

$$r_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 1 \end{bmatrix}, r_2 = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}, r_3 = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}, r_4 = \begin{bmatrix} 1/2 \\ 1/2 \\ 1/2 \\ 1/2 \end{bmatrix}$$

$$\mathcal{R}^{\mathcal{N}_{4,1}^{\text{line}}} = \text{conv}(r_1, r_2, r_3, r_4).$$

$$\mathcal{R}_{\text{framed}}^{\mathcal{N}_{4,1}^{\text{line}}} = \text{conv}(r_1, r_2, r_3).$$

Algorithm results: multihop line network

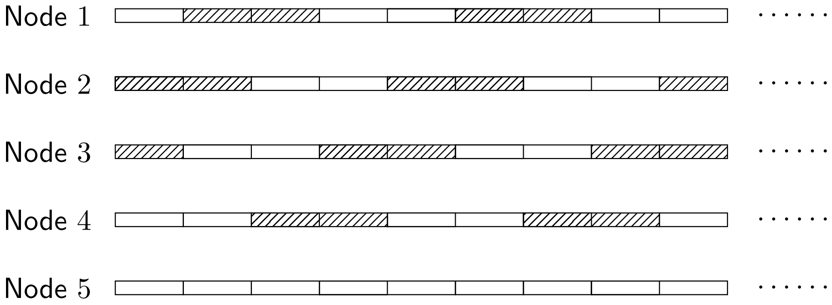


Figure: Node transmitting and receiving states in $\mathcal{N}_{4,1}^{\text{line}}$.

Concluding Remarks

- Provided a fundamental theoretic guidance for network scheduling research, and may motivate many further researches.
- The essential problem: independent sets of a periodic graph.
- Connect periodical independent sets with cycles in scheduling graphs.
- Simplifying the computation costs of enumerating cycles by exploring dominance property.