Reliable Throughput of Generalized Collision Channel without Synchronization

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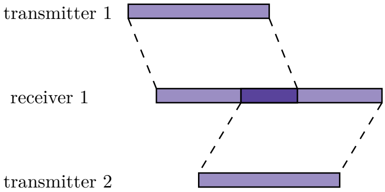
Collision Channel

- Practical communication systems: often multi-user in nature.
- Signals intended for a receiver may cause interference at other receivers.
- Collision channel: the simplest and nontrivial multi-user system
  - Collision: two or more packets overlap at receivers.
- Massey and Mathys [3] studied the collision channel model for multiple access communication under unknown time offsets.



Time offsets

- Time offsets can model both transmission delays and time unsynchronization.
- The work [2] and its seminal research studied communication with fixed time offsets.
- Collision channel: communication under unfixed time offsets.

Protocol Sequence

- Transmissions are under the guidance of protocol sequences.
  - Binary sequences with finite length;
  - Shift-invariant (SI): throughputs remain the same under arbitrary time offsets.
- Massey and Mathys [3] proved that reliable throughputs for multiple access systems can be achieved by SI sequence.
- Families of protocol sequences for multiple access communication: shift-invariant sequences [4], the Wobbling sequences [5], the Chinese Reminder Theorem sequences [1].

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Generalized Collision Channel Model

- A communication system consists of M transmitters $u_1, u_2, \ldots, u_M$, paired up with M receivers $r_1, r_2, \ldots, r_M$.
- Each pair (u_i, r_i) is defined as a link l_i, $i = 1, 2, \ldots, M$.
- In practice, some transmitters and receivers may correspond to an identical physical node.
- $\mathcal{L} := \{l_1, l_2, \ldots, l_M\}$ is the collection of all links.

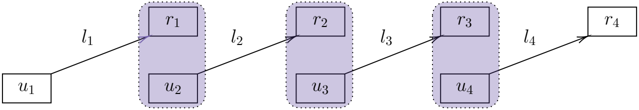


Figure: Line network. Receiver r_i and transmitter u_{i+1} correspond to one physical node, depicted as one shaded area.

Time offsets of Collision Channel

- Time offset δ_k^i: transmitter u_i starts transmission at time t, receiver r_k starts to detect u_i's signal at time $t + \delta_i^k$.
- Collision occurs at receiver r_k as signals from u_i and u_j overlap.

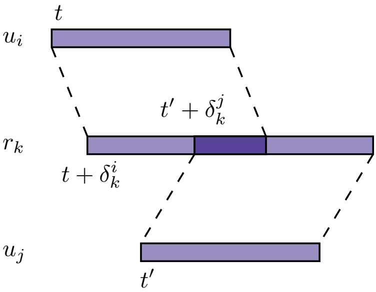
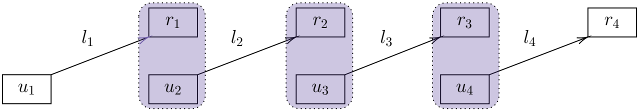


Figure: The time offset δ_k^i is from transmitter u_i to receiver r_k. The dark area at r_k represents a collision.

Collision Profile

Assume signals can propagate two hops at most.



- Since the signals from transmitter u_3 can be detected at receiver r_4, the link $l_3 = (u_3, r_3)$ is in l_4's collision set $\mathcal{I}(l_4)$.
- Signals from u_1 and u_2 cannot reach r_4. Hence, l_1 and l_2 are not in $\mathcal{I}(l_4)$ and $\mathcal{I}(l_4) = \{l_3\}$.
- $\mathcal{I}(l_1) = \{l_2, l_3, l_4\}, \mathcal{I}(l_2) = \{l_1, l_3, l_4\}$ and $\mathcal{I}(l_3) = \{l_2, l_4\}$.
- Collision profile $\mathcal{I} = \{\mathcal{I}(l_1), \mathcal{I}(l_2), \mathcal{I}(l_3), \mathcal{I}(l_4)\}$

Protocol Sequence

- Timeslot $n \in \mathbb{Z}$: the semi-open time interval $n \leq t < n+1$.
- Protocol signal $s_i(t)$ assigned to transmitter u_i: l_i is active if $s_i(t) = 1$ and inactive if $s_i(t) = 0$.
- Continuous protocol signal $s_i(t)$ can be equivalently represented by a binary protocol sequence $s_i := [s_i(0), s_i(1), \ldots, s_i(L_i - 1)]$.

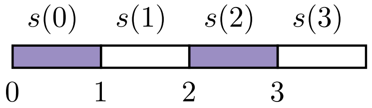


Figure: The protocol signal corresponding to the protocol sequence $\mathbf{s} = [1, 0, 1, 0]$. The shaded areas represent active time intervals.

Reliable Throughput Region

- Duty factor f_i for $\mathbf{s}_i$: the fraction of its nonzero period $f_i = \frac{1}{L_i}\sum_{t=0}^{L_i-1} s_i(t)$.
- Approachable throughput vector $\mathbf{T} = [T_1, \ldots, T_M]$: $\forall \epsilon > 0$, there exist protocol signals $s_i(t)$ for each transmitter u_i, such that the receiver r_i is able to receive correctly the packets from u_i at a rate no smaller than $T_i - \epsilon$ packets/timeslot, for any values of the time offsets.
- We call the collection of all approachable reliable throughput vectors reliable throughput region.

Two Reliable Throughput Regions

- Slot-synchronized cases $\mathcal{C}_s$: the collection of all approachable throughput vectors under arbitrary integer time offsets.
- Non-synchronized cases $\mathcal{C}_u$: the collection of all approachable throughput vectors under arbitrary time offsets.

Major Problems

- What is the difference in reliable throughput region between the slot-synchronized cases $\mathcal{C}_s$ and the non-synchronized cases $\mathcal{C}_u$?
- How to construct protocol sequence to approach reliable throughputs for generalized collision channel?
- How to approach the outer boundary of reliable throughput regions?

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Main Result

Theorem

Given a link set $\mathcal{L} = \{l_1, \ldots, l_M\}$ *and its collision profile* $\mathcal{I}$, $\mathcal{C}_s = \mathcal{C}_u$, *consisting of throughput vectors* $\mathbf{T} = [T_1, \ldots, T_M]$ *such that*

$$T_i = f_i \prod_{j: l_j \in \mathcal{I}(l_i)} (1 - f_j),$$

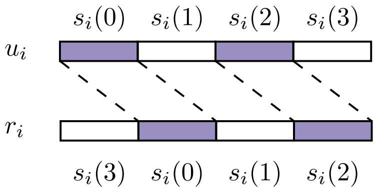
where $\mathbf{f} = [f_1 \ldots, f_M]$ *is a duty factor vector in* $[0, 1]^M$.

Shifted Protocol Sequences

- Under integer time offsets, the receivers receive packets according to a row-wise shifted protocol sequence.
- Transmitter u_i send signals according to $[s_i(0), s_i(1), \ldots, s_i(L-1)]$. Receiver r_j receives $[s_i(-\delta_i^j \bmod L), s_i(1-\delta_i^j \bmod L), \ldots, s_i(L-1-\delta_i^j \bmod L)]$.

Example

Suppose transmitter u_i sends $\mathbf{s}_i = [1,0,1,0]$ and $\delta_i^j = 1$. The signals received at r_i is indeed $[0,1,0,1]$.



Protocol Matrices in Generalized Collision Channel

- Protocol matrix $\mathbf{S} = [s_1, s_2, \ldots, s_M]^\intercal$.
- Each receiver r_i receives a row-wise shifted submatrix $\mathbf{S}^i[\delta]$.
- Submatrix $\mathbf{S}^i[\delta]$: formed by a subset of $\{s_1, s_2, \ldots, s_M\}$.

Example

Suppose $\mathcal{I}(l_2) = \{l_3\}$. Let the time offsets from u_2 to r_2 be 1 and from u_3 to r_2 be 3. When the protocol matrix $\mathbf{S} = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix}$, the row-wise shifted submatrix $\mathbf{S}^2[\delta]$ received by r_2 is

$$\begin{bmatrix} s_2(3) & s_2(0) & s_2(1) & s_2(2) \\ s_3(1) & s_3(2) & s_3(3) & s_3(0) \end{bmatrix} = \begin{bmatrix} 0 & \mathbf{1} & 1 & 0 \\ 1 & 0 & 1 & 0 \end{bmatrix}.$$

Shift-Invariant (SI) Protocol Sequences

- A set of sequences $\{s_1, s_2, \ldots, s_M\}$ is SI if the resulting throughputs $T_1, T_2, \ldots, T_M$ remains the same under arbitrary time offsets.
- The sequence set $\{s_1, s_2, \ldots, s_M\}$ is SI if and only if, in the corresponding protocol matrix $\mathbf{S}$, the combination of columns remains the same under arbitrary row-wise shift.

SI Protocol Sequences in Generalized Collision Channel

- Multiple access communication: **S** is shift-invariant.
- Generalized collision channel: $\mathbf{S}^i$ for each receiver r_i is shift-invariant.
- Are the sequence subsets $\{s_i\}_{i \in J(j)}$ still shift-invariant for all receiver r_j, $j = 1, 2, \ldots, M$?

Lemma

For any integer time offsets δ and collision set $\mathcal{I}(l_i)$ at receiver r_i, $i = 1, 2, \ldots, M$, the combination of columns in $\mathbf{S}^i[\delta]$ remains the same under arbitrary row-wise shift. All constructed $\mathbf{S}^i$ satisfies $T_i = f_i \prod_{j: l_j \in \mathcal{I}(l_i)} (1 - f_j)$.

Non-synchronized Case $\mathcal{C}_u$

- Adjust constructed protocol sequences by replacing 0 with 0^k and 1 with 1^{k-1}.
- The positive integer k is sufficiently large, 0^k and 1^{k-1} denote a string of k zeros and a string of $k-1$ ones.
- All the throughput vectors are approachable using the adjusted protocol matrices for non-synchronized collision channels.

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Outer Boundary

- Outer boundary of reliable throughput region:
  the set of all approachable $\mathbf{T}$, such that there does not exist another approachable $\mathbf{T}'$ with $\mathbf{T} < \mathbf{T}'$.
- Each $T_i = f_i \prod_{j:l_j \in \mathcal{I}(l_i)} (1 - f_j)$ is defined by duty factors.
- Protocol sequences are constructed based on duty factors.
- Find duty factor vectors $\mathbf{f}$ that map to the throughput vectors on the outer boundary.

Characterization of Outer Boundary

- $\mathbf{F} = \text{diag}(\mathbf{f})$.
- $\mathbf{I}$ is the $M \times M$ identity matrix.
- $\mathbf{E} = [e_{ij}] \in \{0, 1\}^{M \times M}$, where $e_{ij} = 1$ if $l_i \in \mathcal{I}(l_j)$ and $e_{ij} = 0$ otherwise.

Theorem

For irreducible $\mathbf{F}(\mathbf{E} + \mathbf{I})$, the duty factor vector $\mathbf{f} \in (0, 1)^M$ determines a point on the outer boundary of a reliable throughput region if only if the Perron–Frobenius eigenvalue of $\mathbf{F}(\mathbf{E} + \mathbf{I})$ is 1.

In the collision channel model for multiple access communication, $\mathbf{E} + \mathbf{I}$ is a matrix of all ones. Then the condition in Theorem. 5 is $\sum_{i=1}^{M} f_i = 1$, consistent to the result in [3].

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