Continuity of Link Scheduling Rate Region for Wireless Networks with Propagation Delays

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Problem Formulation

Aligned Discrete Scheduling

Approximation by Rational Models
Background
- Data frame length is much longer than the propagation delay.
- Without delay: a collision occurs if two nodes transmit simultaneously.
### Signal Propagation Delay vs Data Frame Size

<table>
<thead>
<tr>
<th></th>
<th>Underwater acoustic</th>
<th>Satellite radio</th>
<th>Cellular radio</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Propagation delay</strong></td>
<td>3.3s</td>
<td>0.12s</td>
<td>0.033ms</td>
</tr>
<tr>
<td><strong>Frame size</strong></td>
<td>10s</td>
<td>0.5s</td>
<td>10ms</td>
</tr>
<tr>
<td><strong>Propagation speed</strong></td>
<td>1.5km/s</td>
<td>$3 \times 10^5$km/s</td>
<td>$3 \times 10^5$km/s</td>
</tr>
<tr>
<td><strong>Transmission range</strong></td>
<td>3km</td>
<td>$3.6 \times 10^4$km</td>
<td>10km</td>
</tr>
</tbody>
</table>
Scheduling can benefit from the propagation delay to improve the throughput hsu2009st, guan2011mac, chitre2012throughput, anjiangi16, bai2017throughput, ma2019hybrid.

For a network consisting of $K$ communication pairs designed in chitre2012throughput,

- The total throughput can be $K$ when delay is considered.
- Without making use of the delays, the total throughput is less than 1.

In other words, the gain by considering delay is at least $K$. 
Problem Overview

- Both slotted chitre2012throughput, tong2016strategies, bai2017throughput and unslotted scheduling anjangi2016unslotted, anjangi2017propagation have been studied when propagation delay is taken into consideration.

- Interference alignment is a special case under the model of this paper.
Problem Formulation
Network Model with Delay

- A network of three nodes labelled by 1, 2, 3, where $d(1, 2) = \sqrt{2}$ and $d(3, 2) = \sqrt{5}$.
- Two links $l_1 = (1, 2)$ and $l_2 = (3, 2)$, which have collision to each other (i.e., $\mathcal{I}(l_1) = \{l_2\}$ and $\mathcal{I}(l_2) = \{l_1\}$)

\begin{center}
\begin{tikzpicture}[scale=0.8]
    \node (n1) at (0,0) {node 1};
    \node (n2) at (2,0) {node 2};
    \node (n3) at (4,0) {node 3};
    \node (t1) at (0,2) {$\Theta_1$};
    \node (t2) at (4,2) {$\Theta_2$};
    \node (t3) at (2,2) {$\Theta_3$};

    \draw[->, dashed] (n1) -- (t1);
    \draw[dashed] (n1) -- (n2);
    \draw[->, dashed] (n2) -- (t2);
    \draw[dashed] (n2) -- (n3);
    \draw[->, dashed] (n3) -- (t3);
    \draw[dashed] (n3) -- (n1);

    \draw[->, thick] (n1) -- (2,1) -- (n2);
    \draw[->, thick] (2,1) -- (n3);
    \draw[->, thick] (n3) -- (4,1) -- (n1);

    \node[below] at (2,0) {$\sqrt{2}$};
    \node[below] at (4,0) {$\sqrt{5}$};
    \node[above] at (2,2) {$\sqrt{2}$};

    \node[above] at (0,2) {time};
    \node[above] at (4,2) {time};

\end{tikzpicture}
\end{center}
• Define the link-wise delay between \( l = (s, t) \) and \( l' = (s', t') \) by \( D(l, l') = d(s, t) - d(s', t) \).

• A network \( \mathcal{N} = (\mathcal{L}, \mathcal{I}, D) \) is a directed, weighted graph:
  - Vertex set \( \mathcal{L} \) represents the communication links.
  - Directed edge set \( \mathcal{I} = (\mathcal{I}(l), l \in \mathcal{L}) \) specifies collision relations among links.
  - Weight matrix \( D = (D(l, l'), l, l' \in \mathcal{L}) \) tells the delays.

In the previous example,
\[
D(l_1, l_2) = \sqrt{2} - \sqrt{5} \quad \text{and} \quad D(l_2, l_1) = \sqrt{5} - \sqrt{2}
\]
• For each link $l$, a schedule $S_l$ is a sequence of disjoint, closed intervals, called the **active intervals** of link $l$. For example:

$$S_l = \{[0, 1], [2, 3], [4, 5], \ldots\}$$

$$S_{l'} = \{[0.4, 1.3], [2.5, 3.3], [4.5, 5.3], \ldots\}$$

• A schedule is also written as the union of these intervals:

$$S_l = [0, 1] \cup [2, 3] \cup [4, 5] \cup \cdots$$

$$S_{l'} = [0.4, 1.3] \cup [2.5, 3.3] \cup [4.5, 5.3] \cup \cdots$$

• A schedule is **collision free** if the Lebesgue measure $\lambda(S_{l'} \cap (S_l + D(l, l')))$ is 0 for all $l$ and $l'$ such that $l' \in \mathcal{I}(l)$.
• A practical communication device cannot transmit signals in arbitrarily short time intervals.
• Assume the length of any active interval is bounded below.
• A schedule $S = (S_l, l \in \mathcal{L})$ is called an $\omega$-schedule if the length of any active interval in $S$ is bounded below by $\omega$.
• The following is a $0.8$-schedule.

$$S_l = \{[0, 1], [2, 3], [4, 5]\}$$
$$S_l' = \{[0.4, 1.3], [2.5, 3.3], [4.5, 5.3]\}$$
Scheduling Rate Region

- For a collision-free schedule $S = (S_l, l \in \mathcal{L})$, the rate vector $(R_S(l), l \in \mathcal{L})$ has

$$R_S(l) = \lim_{T \to \infty} \frac{\lambda(S_l \cap [0, T])}{T}.$$ 

- $R_S(l)$ is the fraction of the time that the link $l$ is active.

- A vector $R$ is $\omega$-achievable if for any $\epsilon > 0$, there exists a collision-free $\omega$-schedule $S$, such that $R_S(l) > R(l) - \epsilon$, $\forall l$.

- The $\omega$-scheduling rate region, denoted by $\tilde{R}(\omega, D)$, is the collection of all the $\omega$-achievable rate vectors.
Major Research Issues of the General Case

- How to characterize the rate region?
  - Continuous schedules
  - General non-zero delays

- How to handle the delay measurement error or delay dynamics?
  - Delays cannot be measured without errors.
  - Delays change over time.

- How large gain can be obtained by considering delays?
Basic Properties

- $\tilde{\mathcal{R}}(\omega, D)$ is **convex** and achievable by periodic scheduling.
- $\tilde{\mathcal{R}}(\omega, D) = \tilde{\mathcal{R}}(\alpha\omega, \alpha D)$ for $\alpha > 0$.

**Theorem (Continuity of $\omega$-Scheduling Rate Region)**

Let $\delta = \|D - D'\|_{\infty}$. For any $\omega > 2\delta$ and $R \in \tilde{\mathcal{R}}(\omega, D)$, there exist $\omega' \in [\omega - 2\delta, \omega]$ and $R' \in \tilde{\mathcal{R}}(\omega', D')$ such that $\|R - R'\|_{\infty} < \frac{2\delta}{\omega}$. 
Aligned Discrete Scheduling
A schedule is called a **discrete schedule with timeslot size** $\Delta$ if the lengths of all active/inactive intervals are multiples of $\Delta$.

A discrete schedule with timeslot size $\Delta$ is said to be **aligned to 0** if any boundary $t$ of an active interval satisfies $t \equiv 0 \mod \Delta$. A discrete schedule aligned to 0 is also called a **slotted schedule**.

A slotted schedule can be represented by a binary matrix.
A delay matrix is said to be discrete if all the entries are multiples of a positive value $\Delta$. Both integer and rational delay matrices are discrete.

The collection $\mathcal{R}(\Delta, D)$ of all such achievable rate vectors by discrete scheduling with timeslot size $\Delta$ is called the discrete rate region.

**Theorem**

*For an integer delay matrix $D$, $\mathcal{R}(1, D) = \tilde{\mathcal{R}}(\omega, D)$ for any $0 < \omega \leq 1$.***
Approximation by Rational Models
Approximation of a network $\mathcal{N} = (\mathcal{L}, \mathcal{I}, D)$ by rounding chitre2012throughput:

- Round $D$ to rational $D'$ such that $\|D - D'\|_\infty \leq 5 \times 10^{-(n+1)}$.
- For any $R \in \mathcal{R}(1, 10^n D')$, there exists $R' \in \tilde{\mathcal{R}}(\omega, D)$ with $\omega \in (0, 10^{-n}]$, such that $\|R - R'\|_\infty < 1$.
- Approximation by rounding works for some cases chitre2012throughput.

**Theorem**

Let $\delta = \|D - D'\|_\infty$. For any $\omega > 2\delta$ and $R \in \tilde{\mathcal{R}}(\omega, D)$, there exist $\omega' \in [\omega - 2\delta, \omega]$ and $R' \in \tilde{\mathcal{R}}(\omega', D')$ such that $\|R - R'\|_\infty < \frac{2\delta}{\omega}$.
Theorem (Dirichlet’s theorem on simultaneous approximation)
Suppose $\alpha_1, \alpha_2, \ldots, \alpha_n$ are $n$ real numbers and $Q > 1$ is an integer, then there exist integers $q, p_1, p_2, \ldots, p_n$ with greatest common divisor (GCD) 1, such that

$$1 \leq q < Q^n \quad \text{and} \quad |\alpha_i - p_i/q| \leq 1/(qQ), \ 1 \leq i \leq n.$$  

Approximation of a network $\mathcal{N} = (\mathcal{L}, \mathcal{I}, D)$ by Dirichlet’s theorem:

- Fix an integer $Q > 1$. There exist integers $q, p_{l,l'}$ for $l, l' \in \mathcal{L}$ such that $1 \leq q < Q|\mathcal{L}|^2$ and $|D(l, l') - \frac{p_{l,l'}}{q}| \leq \frac{1}{qQ}$.
- For any $R \in \mathcal{R}(1, qD')$, there exists $R' \in \tilde{\mathcal{R}}(\frac{Q-2}{qQ}, D)$ such that $\|R - R'\|_\infty < 2/(Q - 2)$. 

Example

Consider the matrix

\[ D = \begin{bmatrix} 0 & \sqrt{2} & \sqrt{3} \\ \sqrt{2} & 0 & \sqrt{5} \\ \sqrt{3} & \sqrt{5} & 0 \end{bmatrix} . \]  

<table>
<thead>
<tr>
<th></th>
<th>5</th>
<th>10</th>
<th>15</th>
<th>20</th>
</tr>
</thead>
<tbody>
<tr>
<td>( Q )</td>
<td>125</td>
<td>1000</td>
<td>3375</td>
<td>8000</td>
</tr>
<tr>
<td>( Q^3 )</td>
<td>123</td>
<td>881</td>
<td>2728</td>
<td>4109</td>
</tr>
<tr>
<td>( q )</td>
<td>174</td>
<td>1246</td>
<td>3858</td>
<td>5811</td>
</tr>
<tr>
<td>( p_2 )</td>
<td>213</td>
<td>1526</td>
<td>4725</td>
<td>7117</td>
</tr>
<tr>
<td>( p_3 )</td>
<td>275</td>
<td>1970</td>
<td>6100</td>
<td>9188</td>
</tr>
<tr>
<td>( 10^{-5} \max_i</td>
<td>\sqrt{i} - p_i/q</td>
<td>)</td>
<td>40</td>
<td>8.8</td>
</tr>
<tr>
<td>( 10^{-5}/(qQ) )</td>
<td>160</td>
<td>11</td>
<td>2.4</td>
<td>1.2</td>
</tr>
</tbody>
</table>
Circular $K$-Pair Network (General $K$-pair?)

- By Dirichlet’s theorem, we can construct more networks with unbounded total throughput gain when considering delay in scheduling.