Continuity of Link Scheduling Rate Region for Wireless Networks with Propagation Delays

Yijun Fan[1], Yanxiao Liu[2] and Shenghao Yang[1]

Jun 20, 2022

IEEE ISIT 2022

[1]School of Science and Engineering, The Chinese University of Hong Kong, Shenzhen, Shenzhen, China

[2]Department of Information Engineering, The Chinese University of Hong Kong, Hong Kong, China

Outline

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Background

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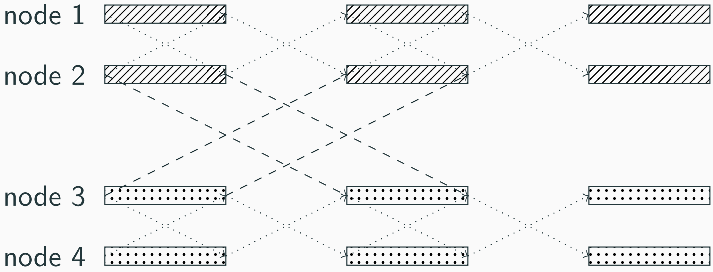
Aligned Discrete Scheduling

Approximation by Rational Models

Background

Traditional Framed Scheduling in Radio Communications

- Data frame length is much longer than the propagation delay.
- Without delay: a collision occurs if two nodes transmit simultaneously.



Signal Propagation Delay vs Data Frame Size

	Underwater acoustic	Satellite radio	Cellular radio
Propagation delay	3.3s	0.12s	0.033ms
Frame size	10s	0.5s	10ms
Propagation speed	1.5km/s	3×10^5km/s	3×10^5km/s
Transmission range	3km	3.6×10^4km	10km

Scheduling with Long Propagation Delay

- Scheduling can benefit from the propagation delay to improve the throughput **hsu2009st**, **guan2011mac**, **chitre2012throughput**, **anjangi16**, **bai2017throughput**, **ma2019hybrid**.
- For a network consisting of K communication pairs designed in **chitre2012throughput**,
  - The total throughput can be K when delay is considered.
  - Without making use of the delays, the total throughput is less than 1.
- In other words, the gain by considering delay is at least K.

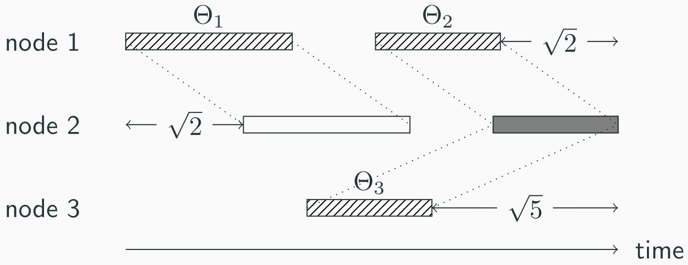
Problem Overview

- Both slotted **chitre2012throughput**, **tong2016strategies**, **bai2017throughput** and unslotted scheduling **anjangi2016unslotted**, **anjangi2017propagation** have been studied when propagation delay is taken into consideration.
- Interference alignment is a special case under the model of this paper.

Problem Formulation

Network Model with Delay

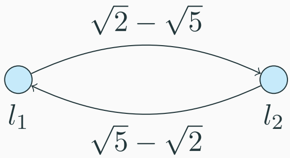
- A network of three nodes labelled by $1, 2, 3$, where $d(1,2) = \sqrt{2}$ and $d(3,2) = \sqrt{5}$.
- Two links $l_1 = (1,2)$ and $l_2 = (3,2)$, which have collision to each other (i.e., $\mathcal{I}(l_1) = \{l_2\}$ and $\mathcal{I}(l_2) = \{l_1\}$)



Link-wise Network Model

- Define the link-wise delay between $l = (s, t)$ and $l' = (s', t')$ by $D(l, l') = d(s, t) - d(s', t)$.
- A network $\mathcal{N} = (\mathcal{L}, \mathcal{I}, D)$ is a directed, weighted graph:
  - Vertex set $\mathcal{L}$ represents the communication links.
  - Directed edge set $\mathcal{I} = (\mathcal{I}(l), l \in \mathcal{L})$ specifies collision relations among links.
  - Weight matrix $D = (D(l, l'), l, l' \in \mathcal{L})$ tells the delays.

In the previous example,
$D(l_1, l_2) = \sqrt{2} - \sqrt{5}$ and
$D(l_2, l_1) = \sqrt{5} - \sqrt{2}$



Scheduling

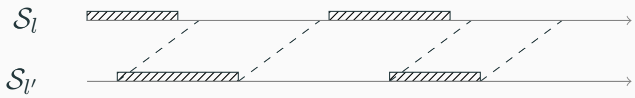
- For each link l, a schedule $\mathcal{S}_l$ is a sequence of disjoint, closed intervals, called the active intervals of link l. For example:

$$\mathcal{S}_l = \{[0,1],[2,3],[4,5],\cdots\}$$
$$\mathcal{S}_{l'} = \{[0.4,1.3],[2.5,3.3],[4.5,5.3],\ldots\}$$

- A schedule is also written as the union of these intervals:

$$\mathcal{S}_l = [0,1] \cup [2,3] \cup [4,5] \cup \cdots$$
$$\mathcal{S}_{l'} = [0.4,1.3] \cup [2.5,3.3] \cup [4.5,5.3] \cup \cdots$$

- A schedule is collision free if the Lebesgue measure $\lambda(\mathcal{S}_{l'} \cap (\mathcal{S}_l + D(l,l')))$ is 0 for all l and l' such that $l' \in \mathcal{I}(l)$.



ω-Scheduling

- A practical communication device cannot transmit signals in arbitrarily short time intervals.
- Assume the length of any active interval is bounded below.
- A schedule $\mathcal{S} = (\mathcal{S}_l, l \in \mathcal{L})$ is called an ω-schedule if the length of any active interval in $\mathcal{S}$ is bounded below by ω.
- The following is a 0.8-schedule.

$$\mathcal{S}_l = \{[0,1],[2,3],[4,5]\}$$
$$\mathcal{S}_{l'} = \{[0.4,1.3],[2.5,3.3],[4.5,5.3]\}$$

Scheduling Rate Region

- For a collision-free schedule $\mathcal{S} = (\mathcal{S}_l, l \in \mathcal{L})$, the rate vector $(R_{\mathcal{S}}(l), l \in \mathcal{L})$ has

$$R_{\mathcal{S}}(l) = \lim_{T \to \infty} \frac{\lambda(\mathcal{S}_l \cap [0, T])}{T}.$$

- $R_{\mathcal{S}}(l)$ is the fraction of the time that the link l is active.
- A vector R is ω-achievable if for any $\epsilon > 0$, there exists a collision-free ω-schedule $\mathcal{S}$, such that $R_{\mathcal{S}}(l) > R(l) - \epsilon$, $\forall l$.
- The ω-scheduling rate region, denoted by $\widetilde{\mathcal{R}}(\omega, D)$, is the collection of all the ω-achievable rate vectors.

Major Research Issues of the General Case

- How to characterize the rate region?
  - Continuous schedules
  - General non-zero delays
- How to handle the delay measurement error or delay dynamics?
  - Delays cannot be measured without errors.
  - Delays change over time.
- How large gain can be obtained by considering delays?

Basic Properties

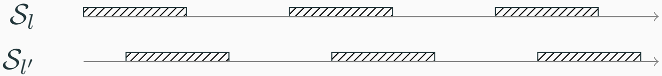
- $\widetilde{\mathcal{R}}(\omega, D)$ is convex and achievable by periodic scheduling.
- $\widetilde{\mathcal{R}}(\omega, D) = \widetilde{\mathcal{R}}(\alpha\omega, \alpha D)$ for $\alpha > 0$.

Theorem (Continuity of ω-Scheduling Rate Region)
Let $\delta = \|D - D'\|_\infty$. For any $\omega > 2\delta$ and $R \in \widetilde{\mathcal{R}}(\omega, D)$, there exist $\omega' \in [\omega - 2\delta, \omega]$ and $R' \in \widetilde{\mathcal{R}}(\omega', D')$ such that $\|R - R'\|_\infty < \frac{2\delta}{\omega}$.

Aligned Discrete Scheduling

Discrete and Slotted Scheduling

- A schedule is called a discrete schedule with *timeslot size* Δ if the lengths of all active/inactive intervals are multiples of Δ.



- A discrete schedule with timeslot size Δ is said to be aligned to 0 if any boundary t of an active interval satisfies $t \equiv 0 \mod \Delta$. A discrete schedule aligned to 0 is also called a slotted schedule.

- A slotted schedule can be represented by a binary matrix.

Discrete Delay Matrices

- A delay matrix is said to be discrete if all the entries are multiples of a positive value Δ. Both integer and rational delay matrices are discrete.
- The collection $\mathcal{R}(\Delta, D)$ of all such achievable rate vectors by discrete scheduling with timeslot size Δ is called the discrete rate region.

Theorem

For an integer delay matrix D, $\mathcal{R}(1, D) = \widetilde{\mathcal{R}}(\omega, D)$ for any $0 < \omega \leq 1$.

Approximation by Rational Models

Approximation of Irrational Delays: Rounding

Approximation of a network $\mathcal{N} = (\mathcal{L}, \mathcal{I}, D)$ by rounding **chitre2012throughput**:

- Round D to rational D' such that $\|D - D'\|_\infty \leq 5 \times 10^{-(n+1)}$.
- For any $R \in \mathcal{R}(1, 10^n D')$, there exists $R' \in \widetilde{\mathcal{R}}(\omega, D)$ with $\omega \in (0, 10^{-n}]$, such that $\|R - R'\|_\infty < 1$.
- Approximation by rounding works for some cases **chitre2012throughput**.

Theorem
Let $\delta = \|D - D'\|_\infty$. For any $\omega > 2\delta$ and $R \in \widetilde{\mathcal{R}}(\omega, D)$, there exist $\omega' \in [\omega - 2\delta, \omega]$ and $R' \in \widetilde{\mathcal{R}}(\omega', D')$ such that $\|R - R'\|_\infty < \frac{2\delta}{\omega}$.

Theorem (Dirichlet's theorem on simultaneous approximation)
Suppose $\alpha_1, \alpha_2, \ldots, \alpha_n$ *are* n *real numbers and* $Q > 1$ *is an integer, then there exist integers* $q, p_1, p_2, \ldots, p_n$ *with greatest common divisor (GCD)* 1*, such that*

$$1 \leq q < Q^n \text{ and } |\alpha_i - p_i/q| \leq 1/(qQ),\ 1 \leq i \leq n.$$

Approximation of a network $\mathcal{N} = (\mathcal{L}, \mathcal{I}, D)$ by Dirichlet's theorem:

- Fix an integer $Q > 1$. There exist integers $q, p_{l,l'}$ for $l, l' \in \mathcal{L}$ such that $1 \leq q < Q^{|\mathcal{L}|^2}$ and $|D(l,l') - \frac{p_{l,l'}}{q}| \leq \frac{1}{qQ}$.
- For any $R \in \mathcal{R}(1, qD')$, there exists $R' \in \widetilde{\mathcal{R}}(\frac{Q-2}{qQ}, D)$ such that $\|R - R'\|_\infty < 2/(Q-2)$.

Example

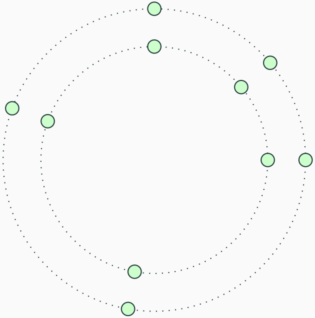
Consider the matrix

$$D = \begin{bmatrix} 0 & \sqrt{2} & \sqrt{3} \\ \sqrt{2} & 0 & \sqrt{5} \\ \sqrt{3} & \sqrt{5} & 0 \end{bmatrix}. \qquad (1)$$

Q	5	10	15	20
Q^3	125	1000	3375	8000
q	123	881	2728	4109
p_2	174	1246	3858	5811
p_3	213	1526	4725	7117
p_5	275	1970	6100	9188
$10^{-5} \max_i \lvert\sqrt{i} - p_i/q\rvert$	40	8.8	0.93	0.086
$10^{-5}/(qQ)$	160	11	2.4	1.2

Circular K-Pair Network **(General K-pair?)**

- By Dirichlet's theorem, we can construct more networks with unbounded total throughput gain when considering delay in scheduling.



circular K-pair network

References i